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If you protest that these are supreme cases, I agree and reply that if you would know an apple, you would not limit your study to the bloom or the little green thing that gives boys colic. You would surely take the mature sample. If you would observe the land, ascend the highest mountain peak and feast your eyes, do not go into some hole in the valley and then say life is just bumping against environment.

The world to-day seems to be short of great leaders. I have wondered whether the philosophers were not to blame. They have been giving us such little, superficial views. There is no depth to life, no reach to vision, just environment. Let us have done with kindergarten views of the nobler qualities of man. Let us invoke the spirit of Spinoza and restate our doctrine of the Absolute One that Spinoza knew as Substance. Let us invoke the spirit of Kant and renew our faith in man, in his moral nature and obedience to the categorical imperative. And then we shall be prepared to invoke the spirit of the supreme philosopher and see reality as Spirit. Then we shall see in its fulness what it is to think.

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ON CERTAIN SUPPOSED FALLACIES OF THE CLASSICAL LOGIC.

No experience is more familiar to the mathematician than the need of extending the primitive meaning of his concepts, so as to include cases not previously suspected. Such extensions of meaning of the mathematician's assumed indefinables or of his symbols of relationship are of the greatest importance. They are only possible to the trained imagination and represent always the most serious obstacles in the learner's progress toward his goal. The mind, which has been only partly trained to follow the paths of scientific rigor, tends to fall back upon its more familiar points of view. It struggles to grasp the new, without yielding up the old conception.

If the mathematician has kept his secrets, as did his early ancestor, the Pythagorean of old, it is chiefly because these secrets have managed to keep themselves. But the distinction between the learner and the learned is, like most distinctions, a relative one. The non-mathematical logician will follow the results which this paper presents to the point, where the demand is made that our implications shall remain true for all meanings of the terms—in particular, when any two terms have been identified. At this point an extension of the meaning of implication will be required and he will drop out of the company of our readers. The mathematical logician, being thus left to go the rest of the way alone, we shall employ the transformations of the class calculus without further explanation. Very soon, however, it will become necessary to extend the meaning of our propositional functions and some of our readers will again be lost to us. Let us hope that a few will find their way through to the end.

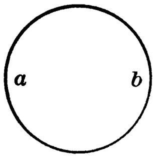
The four propositional functions, which we shall have to employ, will be abbreviated as follows. Let

$\alpha\ (ab) =$ All *a* is all b,
$\beta\ (ab) =$ Some *a* is some *b*,
$\gamma\ (ab) =$ All *a* is some *b*,
$\epsilon\ (ab) =$ No *a* is *b*,

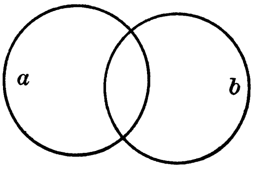
a and *b* standing for class-terms and the word *some* being interpreted to mean *some at least, not all.*

In order to attach a *preliminary* meaning to these forms, let us employ the geometrical analogy invented by Euler. Our four propositions would then be rendered by the diagrams given below.

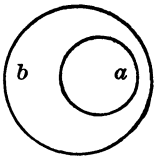
All *a* is all *b*



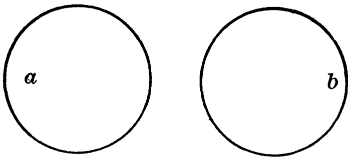
Some *a* is some *b*



All *a* is some *b*



No *a* is *b*



Because they possess similar properties, it is the custom to represent *inclusion* and *implication*—just as the negation of a class and the denial of a proposition—by the same sign, i. e.,

$$a \angle b = a \text{ is included in } b,$$
$$a \angle b' = a \text{ is included in non-b},$$
$$(a \angle b)' = a \text{ is not included in } b, \text{ etc.}$$

Suppose that we should wish to express the propositional functions, α, β, γ and ϵ in the forms that are ordinarily employed. It would then seem natural to recognize the following identities (the multiplication sign, understood but not expressed, between the brackets, standing for the word *and*):

$$\alpha\ (ab) = (a \angle b)\ (b \angle a),$$
$$\beta\ (ab) = (a \angle b)'\ (b \angle a)'\ (a \angle b')',$$
$$\gamma\ (ab) = (a \angle b)\ (b \angle a)',$$
$$\epsilon\ (ab) = (a \angle b').$$

We might then inquire if these representations verify all of the implications, which the traditional logic would set down as true. In order to make this verification complete, it would be enough to deduce the characteristic inferences of the system by means of the transformations of the class calculus. Thus we should have:

$$(1)\ (c \angle b)\ (b \angle a) \angle (c \angle a),$$
$$(2)\ (a \angle b)\ (b \angle c) \angle (a \angle c).$$

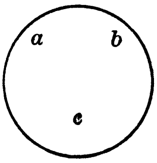
Multiplying together both sides of (1) and (2) and rearranging the factors conjoined in the antecedents,

$$\{(b\angle a)\ (a\angle b)\}\ \{(c\angle b)\ (b\angle c)\}\angle\{(c\angle a)\ (a\ \angle c)\}$$

and this by definition would be the same as

$$\alpha\ (ba)\ \alpha\ (cb)\ \angle\ \alpha\ (ca)$$

(see fig. below)



But it would soon be discovered that some of the implications, that would normally be recognized as valid, break down. Thus, we should expect to have

$$\gamma\ (ab)\ \epsilon\ (ab)\ \angle\ o,$$

(the symbol, o, standing for the *null*-proposition) and this would be rendered by

$$(a\angle b)\ (b\angle a)'\ (a\angle b')\ \angle\ o,$$
$$\text{or } (a\angle b)\ (a\angle b')\ \angle\ (b\angle a)$$

and this result is manifestly not true in general.

Early in the development of the class calculus fallacies similar to the one just noticed were pointed out among the implications of the common logic and it has been the habit of logicians to assert that the relationship of subalternation and some of the valid moods of the syllogism break down. This misapprehension—for it will turn out to be such—is all but universally shared by recent writers. Its removal may be effected by a solution similar to the one which follows.

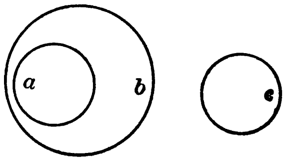
Our rendering of the four propositional functions has been *over-simplified*. Let us attach to them, not the meaning which they had above and which has proven insufficient, but the one which follows, viz.,[1]

[1] For the suggestion which led to this solution I am indebted to Prof. E. A. Singer, Jr. See the article, *Note on the Relation of Subalternation*, in the *Journal of Philosophy*, Sept. 11, 1919.

$$\begin{aligned}
&\alpha\ (ab) = (a \angle b)\ (b \angle a),\\
&\beta\ (ab) = (a \angle b)'\ (b \angle a)'\ (a \angle b')'\\
&\gamma\ (ab) = (a \angle b)'\ (b \angle a)'\ (A' + B),\\
&\epsilon\ (ab) = (a \angle b')A'B'
\end{aligned}$$

where $A = a \angle a'$ and $B = b \angle b'$.

Consider the syllogism, $\gamma\ (ab)\ \epsilon\ (cb) \angle \epsilon\ (ca)$, see fig. below)



We have

$$\begin{aligned}
&(1)\ \ (b \angle a)'B'A'C' \angle A'C',\\
&(2)\ \ (a \angle b)\ (b \angle c') \angle (a \angle c')
\end{aligned}$$

Multiplying together both sides of (1) and (2), factoring and strengthening the antecedent, and remembering that

$$(A' + B)B'C' \angle A'C',$$

there results

$$(a \angle b)\ \ (b \angle a)'\ (A' + B)\ \ (b \angle c')B'C' \angle (a \angle c)A'C',$$
$$\text{or } \gamma\ (ab)\ \epsilon\ (cb) \angle \epsilon\ (ca).$$

since

$$(b \angle c') = (c \angle b') \text{ and } (a \angle c') = (c \angle a').$$

It will be easy to see that the characteristic features of the system, which the common logic would recognize (a system whose implications might be verified empirically by the aid of Euler's diagrams), are now retained.

It must be observed, however, that the interpretation of our four propositional functions, which has just been given, is not unique. All of the implications of the same logic will hold, if the following meaning were to be assigned to the four forms.

$$\begin{aligned}
&\alpha\ (ab) = (a \angle b)\ \ (b \angle a)\ \ (AB + A'B'),\\
&\beta\ (ab) = (a \angle b)'\ (b \angle a)'\ \{(a \angle b')' + (A + B)\},\\
&\gamma\ (ab) = (a \angle b)\ \ \{(b \angle a)'\ (A' + B) + A'B\},\\
&\epsilon\ (ab) = (a \angle b')\ A'B'.
\end{aligned}$$

Let one further illustration suffice:

$$
\begin{aligned}
&(1)\ A'B'C'(a \angle b)'(c \angle b)' \angle B' \\
&(2)\ (b \angle a)\,(a \angle c')\,(b \angle c) \angle B
\end{aligned}
$$

Multiplying together (1) and (2),

$$
\begin{aligned}
&(b \angle a)\,(a \angle c')\,(b \angle c)\,(a \angle b)'(c \angle b)'A'B'C' \angle o, \\
\text{or }\ &(b \angle a)\,\{(a \angle b)'\,(A + B') + AB'\} \\
&(b \angle c)\,\{(c \angle b)'(C + B') + CB'\} \\
&(a \angle c')A'C' \angle o,
\end{aligned}
$$

and this by definition is the same as

$$
\begin{aligned}
&\gamma\,(ba)\,\gamma\,(bc)\,\epsilon\,(ca) \angle o, \\
\text{or }\ &\gamma\,(ba)\,\gamma\,(bc) \angle \epsilon'\,(ca).
\end{aligned}
$$
[2]

Our conclusion may now be briefly stated. In translating the categorical forms of the ordinary logic into the forms of inclusion of the class calculus, we must be careful that our rendering shall not be over-simplified and as a result some of the varieties of immediate inference and of the syllogism be mistakenly regarded as fallacious. A rendering of the propositional functions of the traditional logic is possible, which will allow all of the implications of that logic to remain true without any restriction whatever being imposed upon the meaning of the terms.

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[2] This mood of the syllogism has been selected for illustration because of its analogy with *Darapti*, a mood ordinarily regarded as fallacious (See, for example, Bertrand Russell's *Introduction to Mathematical Philosophy*, p. 164).